Another Dimension

A. TITLE OF LESSON – This three day lesson for Algebra I, part I students is designed to give concrete experiences in finding volume and surface area of prisms, and the volume of cylinders. Students will work with manipulatives, applets, and equations to be able to calculate of how changes in one attribute of the figure will affect surface area and volume.

B. CONTEXT OF LESSON – This lesson would follow behind instruction of VA 8.7A which is the study of surface area and volume of a number of different figures. Students must have mastered:

* the calculation of volume and surface area with the use of formulae
* basic manipulation skills and understanding of equations
* the concepts of volume and surface area

C. LEARNING OBJECTIVES and ASSESSMENT:

|  |  |  |
| --- | --- | --- |
| Learning Objective | Bloom | Assessment (Formative/Summative) |
| To be able to calculate the effect of a dimension change on surface area of a prism.  | Knowledge/Analysis | Activity One Sheet - formative |
| To be able to calculate the effect of a dimension change on volume of a prism. | KnowledgeAnalysis | Activity One Sheet – formativeFill ‘Er up #4 - formative |
| To be able to calculate the effect of a dimension change on the volume of a cylinder. | AnalysisApplication | Lab sheet #3 – formativeObservation of activity 4 (see checklist within lesson) - formative |
| To be able to calculate the effect of a dimension change on surface area/volume of a prism and on volume of a cylinder. | Application | Summative – Evaluation worksheet |

D. RELATED 2009 VIRGINIA STANDARDS OF LEARNING

Sol Objective –VA. 8.7 The student will

b) Describe how changing one measured attribute of the figure affects the volume and surface area. *(Focus of this lesson is on prisms and cylinders)*

E. MATERIALS NEEDED(Provided by teacher)

Clear tape

White 8.5 X 11 paper, many per group

Measuring cups

Access to calculators and computers.

Rulers

Paper Plates

Worksheets (see attached files)

Fill “Er Up (copy only page 1 of the file)\*

Recording Sheet\*

Activity One Sheet

Lab Sheet 3

Another dimension.ppt

Evaluation Worksheet

G. PROCEDURE

Day One

Opening activity (15 minutes)

Prior to starting this activity, arrange the students into pairs and give each group two pieces of paper, tape, Ziploc bag containing unpopped popcorn.

**Teacher – “Each group should do the following. Using an 8.5 X 11 in white paper, turn the paper lengthwise. Curve the paper so that you create a cylinder. Use tape to secure the edges, trying to not overlap the paper. On the second piece of paper, turn the paper widthwise. Curve the paper so that you create a cylinder. Use tape to secure the edges, trying to not overlap the paper. Stand each cylinder on its end. “**

(Students work on this task)

**Teacher -“ You have two cylinders in front of you. How are they different?”**

Possible answers…one is taller than the other, one is wider than the other… accept all answers and make a list of them on the board.

**Teacher – “You have studied figures this year. What is one idea about this figure that you have studied?”**

Possible answers – height, width, bases, faces, surface area, and volume. If volume and surface area are not mentioned, prompt with asking about what you call the inside space and the outside size.

**Teacher – “ For this activity, we are going to focus on surface of these two cylinders. Observe your two cylinders and decide which has a larger volume and by what percent. Discuss your observations with your partner and then come to a consensus.”**

Pairs work to reach consensus.

**Teacher - “Write your conclusion on this recording sheet. Now get with another group. Share both of your conclusions. Discuss them and reach consensus. Write this conclusion on the bottom of both recording sheets. Bring the sheet to me.”**

Read the slips out loud to the class.

**Teacher - “How were the conclusions the same?”**

Accept all answers. Record on board.

**Teacher – “How were the conclusions different?”**

Accept all answers. Record on board.

**Teacher - “Any other observations?”**

Record on board.

**Teacher - “Now is the time to test out your conclusions. Fill each cylinder to the top with popcorn. Try to get the top as level as possible. Then dump out each cylinder individually and measure how much popcorn is in each cylinder. Record the amount on your lab sheet. Then calculate the percent of popcorn the smaller is of the larger. You may use your calculator to help you.”**

Students work on task. Circulate around room to assist as needed.

**Teacher – “Now let’s record our results in this chart.”**

Record each groups results in chart found on the PowerPoint slide.

**Teacher - “What did you notice? Did our results match our predictions? Was anything surprising? “**

Accept all responses.

**Teacher - “What was the main difference between our two cylinders?”**

Possible responses. They were different sizes… etc.

**Teacher - “The difference in size means they had different dimensions. Does anyone know what the word dimension means?”**

Possible responses… different sizes, different heights, different widths.

**Teacher - “These figures have three dimensions. They are referred to as 3-D shapes. Some of you may have heard of 3-D movies where you have to use special glasses to watch the movie. Has anyone been to a 3-D movie and could share what is different about it?”**

Possible answers. The action seems to be real, objects seem touchable, and characters come right off of the screen.

**Teacher - “Yes, 3-D movies seem real because real life is three dimensional. All of us have width, length and height. In this lesson we are going to see how changing the dimensions of figures changes the surface area and volume of that shape. “**

(Instruct the students to pour the popcorn back into the bags and close. Then put the bags and other materials into the large bag provided. Pass out to each pair the following materials: cubes, activity 1 sheet

Activity One. (30 minutes)

**Teacher - “During this activity you will be making cubes from the unifix cubes and finding the surface area and volume of each. We will work the first problem together, and then you will work with your partner to find the answers to the other problems on the activity 1 sheet.”**

Problem One A prism with dimensions of 3X3X3. What is its surface area and volume?

 Surface area\_\_\_\_\_\_\_\_\_\_\_\_\_\_ Volume \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

**Teacher - “Take your cubes and build this prism. How can we find the surface area of this prism?”**

Possible answers – count the number of square on each face and add them together, count the number of squares on each face and multiply by 6 (since it is a cube), count the number of row and multiply by how many rows…

(Be sure solutions are accepted and acceptable. Challenge incorrect/incomplete solutions.)

**Teacher - “Find the surface area of this prism.”**

(Pause to give students time to complete this)

 **“What answer did you get?”**

(Check for accuracy. Correct answer – 54 units squared Have responding student explain how they got that answer.)

**Teacher – “Find the volume of this prism. Guide incorrect/incomplete solution. What answer did you get?”**

(Check for accuracy. Correct answer – 27 units cubed. Have a responding student explain how they got that answer.)

**Teacher – “ Now you are to work with your partner and complete the rest of this worksheet. Create each of the prisms with the cubes and then answer the questions. If you have any questions as a team, raise your hand and I will come and help you.”**

Students will continue to work in pairs on the assignment. At the end of class, collect papers to review. Remediate as necessary before continuing to the next lesson.

Day Two.

Activity Two (20 minutes)

<http://illuminations.nctm.org/ActivityDetail.aspx?ID=6>

Pass out Fill ‘er up lab sheet.

**Teacher – “Right now you are going to complete an activity focusing just on volume of a rectangular prism. To help you complete this activity, please open your browser to the following link** <http://illuminations.nctm.org/ActivityDetail.aspx?ID=6>

**The applet works in this way. (Demonstrate how to build a prism according to a certain set of dimensions and how to find the volume.)**

**Use this applet and work with your partner to complete the Lab sheet. “**

(Circulate and assist as needed)

Collect lab sheets (Hold till the end of the lesson)

**Teacher – “Now we are going to continuing looking at changes in one dimension and how it affects the volume and surface area. I have returned your lab sheet from yesterday. Who will share question 6?”**

(Possible answers include: it increases by 4 units, it increases by one layer, it increases by L∙W. If no one mentioned the last possibility, use the model to show how the one unit increase forms an additional layer on the prism)

Students share and comment on the responses from yesterday.

**Teacher - “Who will share question 5?”**

Students share and comment on the responses from yesterday.

(Possible answers include: it gets larger, it increases by 8,)

**Teacher - “You have all noticed a pattern in how the surface area changes. We are going to look at finding an equation that would fit increases for surface area and volume. We will start with problem 2. What did you get for the volume?”**

Answer - 12 units cubed

**Teacher - “Who remembers the formula for volume?”**

Answer L∙W∙H

**Teacher - “So if we substitute our values into the formula we get 2**∙**2**∙**3, correct?”**

Answer yes. If different answers are given, work problem out on board.

**Teacher – “ What if we increased the height by 1 unit? What would be the new expression and new answer?”**

Answer. 2∙2∙4 = 12

**Teacher - “So what if we didn’t know the value of the increase, could we find an equation we could use. Let’s see what we already know.**

(Write

 V=L∙W∙H

on the board)

**So if we increase the height by X amount, how could we show that in an expression?”**

Answer H+X

**Teacher - “So if we substitute H+X in the equation, what do we get ?”**

Write on board.

**L** ∙ **W** ∙ **(H+X)**

**LWH + LWX** (after using the distributive property)

(If students are unclear, remind them how to use the distributive property.)

**Teacher – “Does that sound reasonable?”**

(Review as necessary to help students understand the substitution)

**Teacher - “Let’s look at surface area. Does anyone remember the formula for the surface area of a cube?”**

2LW +2LH + 2WH

**Teacher – “ If we increase the height by X amount, how could we show that in an equation?”**

Answer H+X

**Teacher – “So if we substitute H+X in the equation, we get ?”**

2LW + 2L(H+X) + 2W(H+X)

2LW + 2LH + 2LX + 2WH + 2WX

**Teacher – “Does that sound reasonable?”**

(Review as necessary to help students understand the substitution)

**Teacher – “Let’s see if it works. If you look at the Activity 1 sheet from yesterday you will see problems 3 and 4. With your partner, work the problem now using the equations that we generated.”**

Students work

**Teacher – “Was it successful?”**

Discuss answers, share work on board if necessary.

(Return Fill ‘Em Up worksheet to students)

**Teacher - “Please look at problem #4 on the Fill ‘Em Up** **worksheet from today. For homework, you are to create an equation for that problem. Be ready to turn in your solution tomorrow.”**

**……………………………………………………………………………………………………………………………………………………………**

(Collect homework from the lesson before. Respond and return to students as soon as possible. Remediate as necessary)

Day 3

Activity 3 (10 minutes)

**Teacher – “Today we are going to take a look at the volume of a cylinder more in depth. To start with, open your browser to**

[**http://www.mrperezonlinemathtutor.com/G/6\_1\_Surf\_Area\_Vol\_Cylinders.html**](http://www.mrperezonlinemathtutor.com/G/6_1_Surf_Area_Vol_Cylinders.html)

**Teacher – “Using this applet, complete lab sheet 3 with your partner. Be ready to share your answers.”**

(Students work on lab sheet 3 in pairs. Circulate and assist as needed)

**Teacher – “What did you observe when you kept the radius constant? “**

(Fill in student responses on PowerPoint slide)

**Teacher – “What did you observe when you kept the height constant?”**

(Fill in student responses on PowerPoint slide)

**Teacher – “How were the changes in dimension different between the prism and the cylinder?”**

(Students answer. Possible responses may include whole numbers, cylinder doesn’t have a clear pattern, etc. The point here is to note the difference between adding a layer (prism) and adding to a circular shape(cylinder). Place the data on board if necessary.

Activity 4 (15 minutes)

**Teacher - “For this activity you are going to use your estimation skills to create a cylinder that will hold 2 cups of rice. Take a sheet of paper and fold it widthwise. Use tape to secure it and fill it with rice. (This is similar to the activity that you completed in the opening activity. ) Measure the rice. Using only paper, scissors, tape and rice, create a cylinder that holds 2 cups of rice. Once you have completed this task, raise your hand and I will come and you can defend your cylinder. “**

**Observation checklist**

* **Does the cylinder hold 2 cups of rice?**
* **How did you arrive at this cylinder?**
* **What would you need to do to get a cylinder to hold one cup of rice?**

( After the group has successfully completed the task, continue on with these instructions)

 **Teacher – “ I will give you a ruler with which you need to measure the dimensions of your cylinder.**

**Then compute the volume using the formula and determine how close to exactly two cups?”**

(After all groups are finished, share results)

Conclusion (10 min)

**Teacher - “Let’s sum up what we have learned over the past three days. (Open PowerPoint summary page) With your partner, discuss each of the review questions. Then find another pair and discuss them again. Then your group of four needs to find another group of four and have the same discussion. Be prepared then to share your findings with the whole group.”**

(Students work on this task. When mega groups are finished begin to complete group summary)

**Teacher – “How did you respond to the first question?”**

Students respond

(Record responses on the PowerPoint slide. Encourage them to share answers including differences in shape and size).

**Teacher – “How did you respond to the second question?”**

Students respond

(Record responses on the PowerPoint slide. Generate a couple of shapes that would even be more difficult to calculate and possibly an easier shape.)

**Teacher – “How did you respond to the third question?”**

Students respond

(Record responses on the PowerPoint slide. Accept all answers. If not mentioned, have them consider shipping items and the cost of materials to create the containers.)

**Teacher – “How did you respond to the fourth question?”**

Students respond

(Record responses on the PowerPoint slide. Further probe – which shapes are still difficult, why? Is Surface Area or Volume more difficult?)

**Teacher – “Now is your chance to show me individually your understanding of dimension change in figures. I am going to hand out an evaluation of your learning for our lessons. You have 25 minutes to select two of the questions to respond to. I will then collect them and grade them.”**

(Collect the papers at the end of the class. Grade and return as soon as feasible. Work with individual students as needed)

……………………………………………………………………………………………………………………………………………………….

G. MEETING THE NEEDS OF ALL STUDENTS: Students in this class have been tracked in math for the past three years. At the eighth grade level, they are just now attempting Algebra in any form. It is likely that their computation skills will not be very strong. Students will need access to calculators and be able to use them at any point. Students should also have the option to use the manipulatives for their work. It is also likely that the pace of this lesson will need to be slowed down for the entire class, particularly if this is the co-op class. If there is a co-op teacher available, he/she will need to go over the lesson plan in advance. Individual students will likely need follow-up after each lesson in the form of individual remediation. I think the key to helping to make this successful is two-fold. First, the student must have mastered the prerequisite material. Second, students should be carefully pair so that students have the maximum opportunity to learn from each other. This includes not only achievement levels but also partner skills which will be crucial for this three day activity.

H. WHAT COULD GO WRONG WITH THIS LESSON AND WHAT WILL YOU DO ABOUT IT?

* Lesson takes longer than anticipated. Materials need to be prepared in advance for the entire 3 day unit. Since the lessons are divided into sections, I will be able to break at any activity chance. If the break changes the homework assignment, then time will need to be allotted into the lesson for students to work on the homework/individual assignment during class.
* This lesson is really messy. I use mats on the desks which helps to keep things on top. Another option would be to use aluminum trays. However, a broom at the end of class is going to be necessary. I also would have extra rice for the pair that breaks/busts/drops their bag on the floor.
* Absences. If the student misses day 1, they could be paired with another pair to complete day 2 and three. However, if a student misses day 1 and 2 then they will need to work on an alternative plan and complete this activity at another time. This would also be true of students who miss either day 2 or day 2/day 3.
* Internet is not working on student computers. Websites should be preloaded onto teacher computer for use on mimeo.
* Internet not working at all. Skip applet sections. It might be possible to move them to a second day depending on when the internet interruption occurs.
* Students having trouble typing in websites. Create a document with links and upload to Google shared folder.

I. CONNECTION TO CTA: I think there were three speakers that affected this particular lesson. The session with the presenters from NCTM was most helpful. I am a member but had never really taken the time to search and look at the Illuminations materials. Their website is not extremely friendly so I was pleasantly surprised to be able to locate additional materials that were useful for this lesson. The presentation by J. Hill actually started me thinking about connect number sentences to algebra sentences in a new way. I think as algebra teachers we tend to go from equation to number sentence without thinking about the benefits of taking a number sentence and translating it into an equation. I incorporated that idea briefly in this lesson. And finally, John Strebe reminded me about how important getting the chance to work with and share with before answering was to classroom climate and learning.

**\*Resources**

Popcorn, Anyone activity from NCTM - Illuminations <http://illuminations.nctm.org/LessonDetail.aspx?id=L797>)

Fill ‘Er Up activity from NCTM – Illuminations

<http://illuminations.nctm.org/ActivityDetail.aspx?ID=6>

****

****

****

****

****

****